These two integrals are not enough to integrate the equation of motion but can help us to comment on the physical sense of the problem. Two examples follow.

Example 1. Let us assume the case when the mass variation is a linear function of time

$$m = m_0(1-\alpha t) \tag{20}$$

where α is a constant value. According to Eq. (19) the damping coefficient must satisfy the relation

$$\delta = 4\alpha/(1+n) \tag{21}$$

When the central gravitation force is inversely proportional to the square of the distance $n=-2$

$$\delta/\alpha = 4/3 \tag{22}$$

and the conservation law is

$$(r\dot{r}+\dot{r}^2 m+2kr^{-1}m)m^{-1/3}+3K^2m^{4/3}r^{-2}/(4\alpha) = C = \text{const} \tag{23}$$

Example 2. The damping coefficient satisfies the relation

$$\delta = -\left(\frac{\mathrm{d}m}{\mathrm{d}t}\right) \tag{24}$$

The trajectory is described in the form

$$\ddot{r} - K^2 r^{-3} = kr^n \tag{25}$$

The first integral is

$$\dot{r}^2 - 2kr^{n+1}/(n+1) + K^2 r^{-2} = \text{const} \tag{26}$$

It corresponds to the results in Refs. 3 and 4.

Conclusion

For the system with variable mass under the influence of a central nonlinear force and linear damping, conservation laws exist. The mass and the reactive force produced by mass variation are functions of time. The system presents a nonconservative system that has a Lagrangian. According to Noether's theorem such a system has conservation laws. In this Note the conservation laws are found. These first integrals have a very important role in considering the dynamic behavior and detecting chaotic motion of the system. The conservation laws presented in this Note are valuable only for strictly defined mass variation. For some simplifications in the system the standard Kepler problem is obtained and the conservation law reduces to the well-known form.

Acknowledgments

The investigations were supported by the government of Serbia, and VANU. It is my great pleasure to express at this point my gratitude to Dj. S. Djukic, who suggested this problem to me and provided me with very helpful advice and guidance.

References

[1]Vujanovic, B. D., Jones, S. E., *Variational Methods in Nonconservative Phenomena,* Academic Press, New York, 1989.

[2]Meshcherski, I. V., *Dinamika tochki peremennoj massji,* M.D., University of Petersburg, 10/XII, 1897 (in Russian).

[3]Vujanovic, B., "On One First Integral of Planetary Motion," *Astronautica Acta,* Vol. 15, No. 4, 1970, pp. 231–233.

[4]Djukic, Dj., "Algebraic Solution of the Kepler Problem Using the Vujanovic Vector, *Astronautica Acta,* Vol. 18 (supplement), 1974, pp. 161, 162.

Evaluation of Missile Seeker Dwell Time for Three-Dimensional Aerial Engagements

C. B. Asthana* and Prahlada†
Research Centre Imarat, Hyderabad 500269, India

Nomenclature

A = circle obtained in plane 1 at the start of detection
a = position of target at the start of detection measured along the radius of A (i.e., along r)
R = range of target from missile at the start of detection (measured along line of sight)
r = radius of A
V = magnitude of $\bar{V}$ (i.e., $|\bar{V}|$)
$\bar{V}$ = relative velocity vector of target with reference to missile
$\bar{V}_n$ = $V\cos\gamma$
$\bar{V}_r$ = $V\sin\gamma$
β = $\tan^{-1}[r/\sqrt{(R^2-a^2)}]$ (i.e., ½ beamwidth)
γ = smaller angle between $\bar{V}$ and plane 1 measured in plane 2
ϕ = angle between the outward radial vector of A containing the target and $\bar{V}_n$, measured in plane 1 (+ve anticlockwise as seen from the missile)
ψ = ½ the angle subtended at the antenna center in plane 2 by the chord of A along $\bar{V}_n$ (i.e., $\angle NOQ$, Fig. 2)

Introduction

SURFACE-to-surface missiles or air-to-air missiles that use mid-course guidance followed by a terminal homing phase need to acquire the target at the start of the homing guidance phase. For acquisition, the onboard radar system would require the target to be in the beam for a finite time. This finite dwell time on target is necessary to process the received signal for detection, confirmation, and subsequent tracking of the target. Hence the problem of determining a circular area perpendicular to the beam axis that accounts for a finite dwell time is important in the context of target acquisition without search in a three-dimensional aerial engagement. Kuno et al.[1] have considered the interaction of the radar beam with a plane that contains the target velocity vector. The intersection results in an ellipse, but for analytical convenience they have approximated it to a circle and pointed out that the error thus introduced would not affect the results for their application. Kouba and Bose[2] have evaluated the pointing angle error circular error probability (CEP) as a function of a unit pointing vector. The pointing angle error that results from the combined effect of all of the error sources is obtained as the root-sum-square of the individual error. Their analysis indicated the relation between the probability of target sighting by the missile and the error in the pointing vector. Kuno et al. and Kouba and Bose, however, did not address dwell time determination procedures. In this Note a three-dimensional engagement is considered and the evaluation of the minimum value of dwell time is represented in the form of a nomogram. Some of the variables are redefined more appropriately here, as compared with those given in Ref. 3. The variables that affect the estimate of the minimum dwell time have been grouped into an optimal set of independent parameters, namely V/R, a/r, γ, and β. The present study is not concerned with determining the seeker dwell time required for detection and confirmation of the tar-

Received June 29, 1992; revision received Jan. 10, 1993; accepted for publication Jan. 18, 1993.

*Scientist, Control System Laboratory.
†Project Director.

get. Instead, it analyzes what engagement parameters would ensure the given dwell time. The utility of this nomogram lies in determining the circular area around the beam axis to ensure the required dwell time for acquisition without search, and then working out the requirements of accuracy for homing subsystems to capture the target in that area. After the formulation for the evaluation of dwell time is given, the selection of values for the independent parameters is indicated. A typical case study is followed by a conclusion.

Formulation for Evaluation of Dwell Time in a Three-Dimensional Engagement

The general three-dimensional engagement case can be considered in two different planes: one is perpendicular to the beam axis (plane 1), the other contains the missile antenna center and the target relative velocity vector (plane 2). The components of the velocity vector, then, can be considered in both planes (Fig. 1). The details of geometry in plane 1 and plane 2 are shown in Fig. 2 and Fig. 3, respectively. The details of the relationships among angles and distances in both planes are explained in Eqs. (1–4).

$$b = a\cos\phi \tag{1}$$

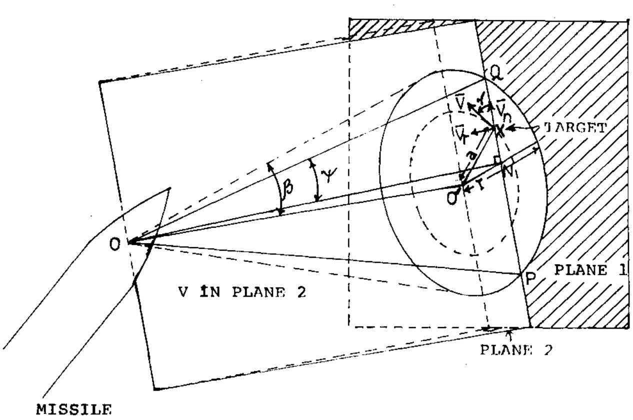

Fig. 1 Missile-target engagement geometry.

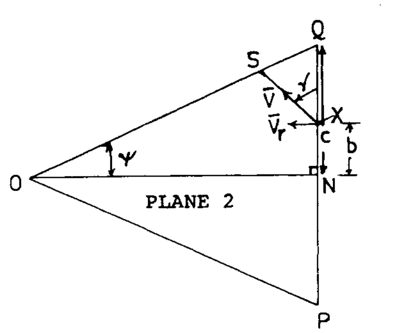

Fig. 2a Geometry in plane 2.

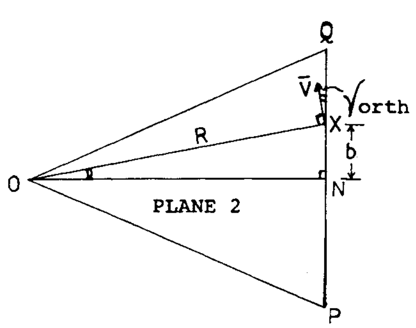

Fig. 2b $\bar{V}\perp$ to range case.

$$ON = \sqrt{R^2 - b^2} = R\sqrt{1-(a/R)^2\cos^2\phi} \tag{2}$$

$$\begin{aligned} c &= \sqrt{(OQ)^2-(ON)^2} = \sqrt{(OO')^2 + r^2 - (ON)^2} \\ &= \sqrt{(R^2-a^2)+r^2-(ON)^2} \\ &= R\sqrt{\tan^2\beta - (a/R)^2(\tan^2\beta + \sin^2\phi)} \end{aligned} \tag{3}$$

$$\tan\psi = c/ON = \sqrt{\frac{\tan^2\beta-(a/R)^2(\tan^2\beta+\sin^2\phi)}{1-(a/R)^2\cos^2\phi}} \tag{4}$$

In the analysis the following assumptions are made:

1) The magnitude and direction of the target velocity relative to the seeker beam do not change during the duration of the dwell time. The assumption implies that the seeker is stabilized against the missile's pitching and yawing motions during acquisition.

2) The target is detectable by the missile seeker even when the target is located at the edge of the beam (i.e., detectable region $0 \le a \le r$). Because it is assumed that the target motion is confined to plane 2, the time t for which it would remain in the seeker beam (i.e., the dwell time) is given by

$$t = XS/V \tag{5}$$

Considering triangle XSQ in Fig. 2a, we get

$$\frac{XS}{\sin\angle SQX} = \frac{QX}{\sin\angle XSQ}$$

Therefore,

$$XS = \frac{(c-b)\sin(90\ \text{deg}-\psi)}{\sin[90\ \text{deg}-(\gamma-\psi)]} = \frac{(c-b)\cos\psi}{\cos(\gamma-\psi)} \tag{6}$$

Using Eq. (6) in Eq. (5) we get

$$t = \frac{(c-b)\cos\psi}{V\cos(\gamma-\psi)} = \frac{(c-b)}{V(\cos\gamma+\sin\gamma\tan\psi)} \tag{7}$$

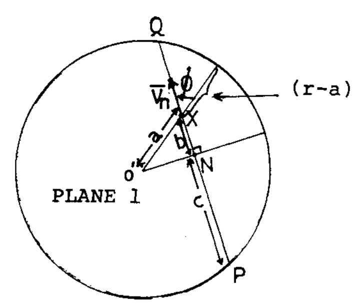

Fig. 3a Geometry in plane 1.

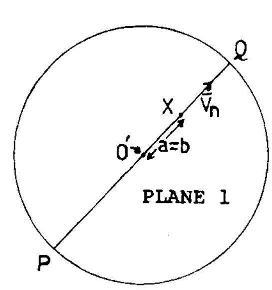

Fig. 3b $\phi = 0$ deg case.

Alternatively,

$$ON \tan \psi = NQ \tag{8}$$

Differentiating both sides with respect to time,

$$\frac{\mathrm{d}(ON)}{\mathrm{d}t} \tan \psi = \frac{\mathrm{d}(NQ)}{\mathrm{d}t}$$

because ψ is constant or,

$$\bar{V}_r \tan \psi = \bar{V}_n' \tag{9}$$

Thus, due to $\bar{V}_r$, $\bar{V}_n'$ is the additional speed along $\bar{V}_n$. This indicates that the shrinkage in the beamwidth due to the closing speed is accounted for. Therefore, if t is the travel time along $\bar{V}_n$, then

$$t = \frac{(c-b)}{\bar{V}_n + \bar{V}_r \tan \psi} \tag{10}$$

Eq. (10) is the same as Eq. (7). Therefore, substituting for b and c from Eqs. (1) and (3) into Eqs. (7) or (10), we get

$$t = \frac{\sqrt{R^2\tan^2\beta - a^2(\tan^2\beta + \sin^2\phi)} - a\cos\phi}{V\left[\cos\gamma + \sin\gamma\sqrt{\dfrac{R^2\tan^2\beta - a^2(\tan^2\beta + \sin^2\phi)}{R^2 - a^2\cos^2\phi}}\right]} \tag{11}$$

It can be noted that for a target appearing at X (Fig. 1) and moving inward (i.e., into the beam), ϕ will be greater than 90 deg. Also, that $\phi = 0$ deg would mean a radially outward moving target and consequently would offer minimum dwell time ($t_{\min}$). Thus setting $\phi = 0$ deg in Eq. (11), we get

$$t_{\min} = \frac{r-a}{V(\cos\gamma + \sin\gamma\tan\beta)}$$

$$= \frac{(1-a/r)}{\left(V/\sqrt{R^2-a^2}\right)(\cos\gamma/\tan\beta + \sin\gamma)} \tag{12}$$

Note that for $\phi = 0$, $c = r$, $b = a$, and $\psi = \beta$. Assuming $R \gg a$, we get

$$t_{\min} = \frac{1-a/r}{V/R(\cos\gamma/\tan\beta + \sin\gamma)} = (1 - a/r)/\underline{X} \tag{13}$$

where

$$\underline{X} = V/R(\cos\gamma/\tan\beta + \sin\gamma)$$

Eq. (13) gives the minimum dwell time and, therefore, has been considered in the development of the nomogram shown in Fig. 4. In the previous expressions only positive square roots are considered.

Selection of Values for Independent Parameters

The radar beamwidth is assumed as 8.0 deg ($\beta = 4.0$ deg). Considering the practical ranges of R and V as $R \geq 5$ km, and $0 \leq V \leq 2000$ m/s, respectively, V/R is allowed to take on values from 0 to 0.2 rad/s. The range of γ is taken as $0 \leq \gamma \leq 90$ deg, and ideally, the range of a/r is 0 to 1.0; but for plots $0.4 \leq a/r < 0.95$ is adequate (Fig. 4). However, because the plots for $\underline{X}$ vs V/R are straight lines they can be extended to cover higher ranges of V/R.

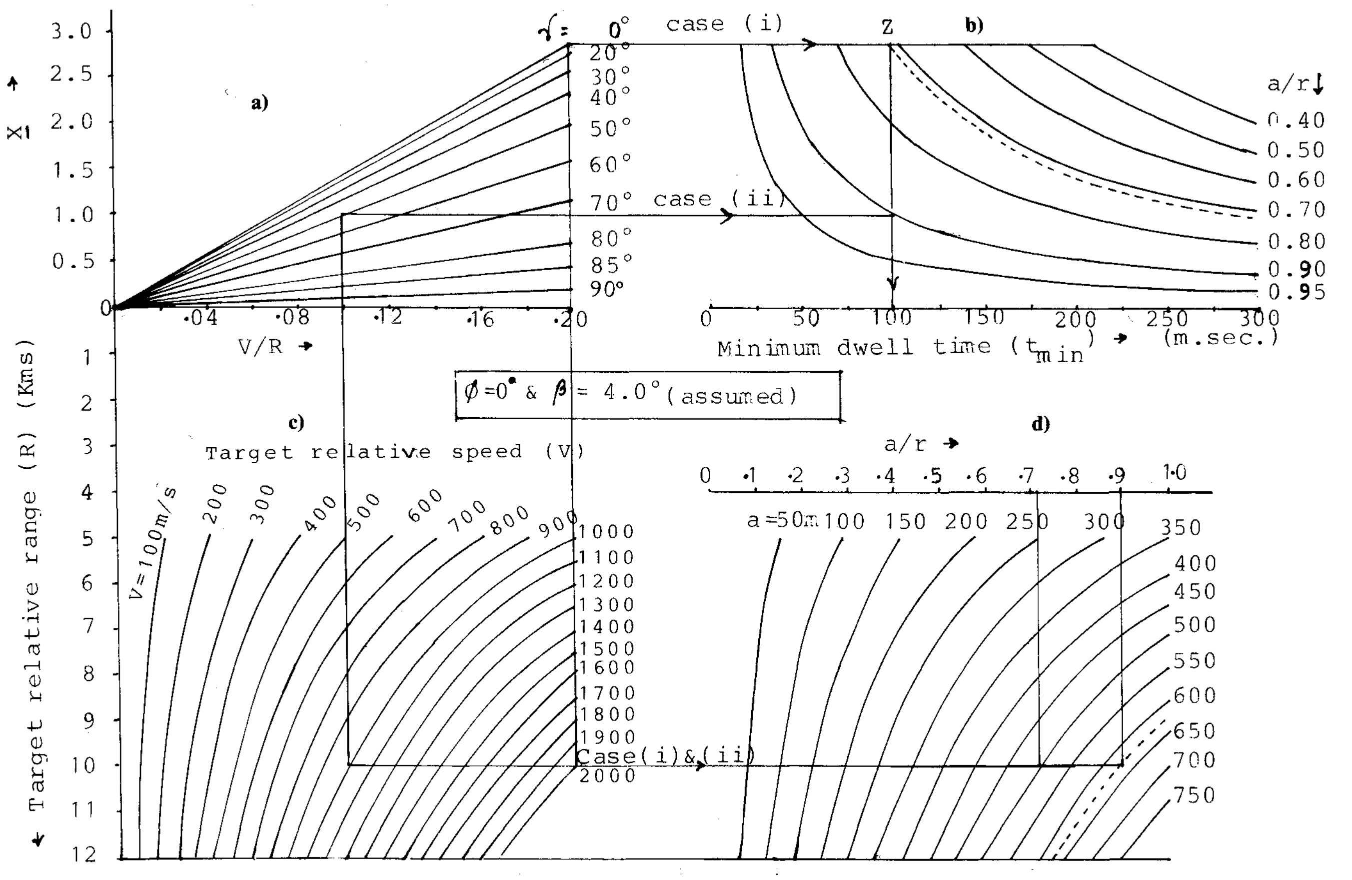

Fig. 4 Nomogram for evaluation of minimum dwell time ($t_{\min}$).

Case Study

Figs. 4a and 4b constitute the nomogram with respect to the optimal set of parameters. Figs. 4c and 4d are helpful in quickly reading the values of individual variables that compose the parameters. From Figs. 4a and 4b it can be seen that for given engagement parameters (i.e., γ and V/R) the minimum dwell time available is controlled by a/r. Three cases are considered here for illustration.

Case 1. Worst Case

Let us assume that the requirement is to ensure a minimum dwell time of 100 ms for a typical encounter geometry and $V/R = 0.2$ rad/s. Thus, starting from $V/R = 0.2$ rad/s in Fig. 4a and $t_{\min} = 100$ ms in Fig. 4b, and following the route shown as case 1, point Z is located in Fig. 4b. The value chosen for γ is 0 because it gives minimum dwell time. At point Z, a/r is 0.71. To find the value of a assume an acquisition range of 10 km. Then, following the case 1 route in Figs. 4c and 4d, for $a/r = 0.71$, $a = 500$ m. A value for V/R less than 0.2, however, would lead to a higher value of a for the same R and $t_{\min}$.

Similarly, any constraint on the lowest value of γ would also result in an increment in a/r and, therefore, in a. It can also be noted here that the chance of $\phi = 0$ deg would also be low, and that for $\phi > 0$, the dwell time would always be more than that read by the nomogram. For $\phi > 0$, Eq. (11) can be used.

Case 2. Typical Case

Let $R = 10$ km, $V = 1000$ m/s, and $\gamma = 50$ deg. In this case the minimum dwell time requirement of 100 ms can be met only for cases with $a/r \leq 0.9$ following the case 2 route from Fig. 4c to 4a to 4b. From Fig. 4d, a is found as 630 m.

Case 3. Zero Closing Speed Case

In this case, V is orthogonal to the R vector (i.e., OX or the seeker line of sight). From Fig. 2b

$$\gamma_{\text{orth}} = \tan^{-1}\left(\frac{b}{ON}\right) = \tan^{-1}\left(\frac{a \cos \phi}{R\sqrt{1-(a/R)^2 \cos^2\phi}}\right)$$

For a radially outward moving target, $\phi = 0$ and then

$$\gamma_{\text{orth}} = \tan^{-1}\left(\frac{a}{\sqrt{R^2 - a^2}}\right) = \tan^{-1}\left[(a/r)\tan\beta\right]$$

In this case, for a target sighted at the beam center (i.e., $a = 0$) $\gamma_{\text{orth}} = 0$; and from Eq. (12)

$$t_{\text{dwell}} = \frac{r}{V} = \frac{\tan\beta}{V/R} = \frac{\text{½ beam width}}{\text{line-of-sight rate}}$$

This would be found in the nomogram by extending graph b in Fig. 4 for the $a/r = 0$ case. It should be noted that when closing speed is zero, Doppler shift is zero. In fact, in this case R is defined as the miss distance. Also, in general, line-of-sight rate = (component of $\bar{V}$ orthogonal to line-of-sight)/R = $[V\cos(\gamma - \gamma_{\text{orth}})]/R$.

Conclusion

A nomogram has been designed to define the circular area within which to find the target for a wide range of missile engagement parameters, given a dwell time constraint.

References

[1]Kuno, H., Nakajima, H., and Ueno, Y., "Mid-Course Guidance for Fire and Forget Missile—Modification of Present Homing Missile," *MEDE Conference Proceedings*, Interania S.A., Geneva, Switzerland, 1979, pp. 597–608.

[2]Kouba, J. T., and Bose, S. C., "Terminal Seeker Pointing-Angle Error at Target Acquisition," *IEEE Transactions on Aerospace and Electronics System*, Vol. AES-16, No. 3, May 1980, 313–319.

[3]Asthana, C. B., and Prahlada, "Evaluation of Missile Seeker Dwell Time for Three-Dimensional Aerial Engagements," *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, AIAA, Washington, DC, 1989, pp. 506–509 (AIAA Paper 89-3484).

Effect of Actuator Coupling on Active Vibration Control of Flexible Structures

Jeffrey B. Horner*
Lockheed Missiles and Space Company, Sunnyvale, California 94089
Daniel J. Rutterman†
Siecor Corporation, Hickory, North Carolina 28603
and
Peter H. Meckl‡
Purdue University, West Lafayette, Indiana 47907

Introduction

THE control of vibration in large structures is complicated by the presence of many closely-spaced modes within the controller bandwidth. Problems of stability robustness arise since accurate models for the entire structure are difficult to generate. This Note investigates stability issues associated with a decentralized control strategy applied to subsystem models of the structure, focusing on the case when subsystem frequencies are closely spaced. Subsystem interaction is assumed to exist primarily because of the actuators, whose locations are usually constrained by physical limitations.

An excellent overview of decentralized control approaches has been written by Sandell et al.[1] One of the first papers to specify conditions for the existence of a local output feedback law was written by Wang and Davison.[2] Conditions on the coupled system dynamics to guarantee stability for decentralized controllers have been developed by Nwokah and Perez.[3] Iftar and Özgüner[4] have proposed a design method for decentralized controllers based on linear quadratic Gaussian/loop transfer recovery (LQG/LTR) techniques.

The work presented in this Note addresses the issue of decentralized control stability for systems having significant subsystem interaction. A novel coupling parameter is introduced that characterizes the degree of subsystem interaction in a 2×2 system that is coupled by the actuator forces. This parameter is used to determine stability bounds for both decentralized and centralized active vibration control. A simple cantilever beam structure with attached disk is used to confirm the theoretical development experimentally.

Stability Analysis

A specific class of structural systems will be considered whose subsystems interact only through the applied forces. The equations of motion for a general structural system model coupled only through the actuators can be represented as follows:

$$I\ddot{Y} + [\diagdown 2\zeta_i\omega_i \diagdown]\dot{Y} + [\diagdown \omega_i^2 \diagdown]Y = PF \tag{1}$$

where $[\diagdown 2\zeta_i\omega_i \diagdown]$ and $[\diagdown \omega_i^2 \diagdown]$ represent diagonal matrices that depend on the damping ratio ζ_i and natural frequency ω_i and P represents a nondiagonal coupling matrix.

Before a suitable stability analysis can be developed, Eq. (1) must be converted into transfer matrix form by taking the

Received April 25, 1992; revision received Feb. 18, 1993; accepted for publication Feb. 19, 1993.

*Engineer, P.O. Box 3504, Mail Stop O-7750, B-579.

†Engineer, 489 Siecor Park.

‡Assistant Professor, 1288 Mechanical Engineering Building.